Constraining Extensions of the Quark Sector with the CP Asymmetry in $B \to \psi K_S$

Galit Eyal* and Yosef Nir†

Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel

WIS-99/28/AUG-DPP

Models with extended quark sector affect the CP asymmetry in the $B \to \psi K_S$ decay, $a_{\psi K_S}$, in two ways: First, the top-mediated box diagram is not necessarily the only important contribution to $B - \bar{B}$ mixing. Second, the 3×3 CKM matrix is no longer unitary. We analyze the constraints that follow from the CDF measurement, $a_{\psi K_S} = 0.79^{+0.41}_{-0.44}$, on the mixing parameters of extended quark sectors. Most noticeably, we find significant constraints on the phase of the relevant flavor changing Z coupling in models with extra down quarks in vector-like representations. Further implications for the CP asymmetry in semileptonic B decays are discussed.

*e-mail: galit@wicc.weizmann.ac.il

†e-mail: ftnir@wicc.weizmann.ac.il

1 Introduction

The CDF collaboration has reported a preliminary measurement of the CP asymmetry in the $B \to \psi K_S$ decay [1]:

$$a_{\psi K_S} = 0.79^{+0.41}_{-0.44}, \tag{1}$$

where

$$\frac{\Gamma(\bar{B}^0_{\rm phys}(t) \to \psi K_S) - \Gamma(B^0_{\rm phys}(t) \to \psi K_S)}{\Gamma(\bar{B}^0_{\rm phys}(t) \to \psi K_S) + \Gamma(B^0_{\rm phys}(t) \to \psi K_S)} = a_{\psi K_S} \sin(\Delta m_B t). \tag{2}$$

(Previous searches have been reported by OPAL [2] and by CDF [3].) Within the Standard Model (SM), this value is cleanly interpreted in terms of the angle β of the unitarity triangle, $a_{\psi K_S} = \sin 2\beta$. In the presence of new physics, this interpretation is modified.

In a previous work with Barenboim [4], we analyzed the constraints that follow from eq. (1) on the size and, in particular, the phase of contributions from new physics to $B - \bar{B}$ mixing. There we investigated models in which the only relevant effect is a new, significant contribution to $B - \bar{B}$ mixing. This large class of models includes, for example, supersymmetric and left-right symmetric extensions of the SM. In particular, we assumed that the $b \to c\bar{c}s$ decay is dominated by the W-mediated tree-level diagram and that the 3×3 CKM matrix is unitary. While the first ingredient holds in almost all reasonable extensions of the SM, the second holds only in models where the quark sector consists of just the three generations of the SM. In this work, we study extensions of the quark sector, namely we relax the assumption that the 3×3 CKM matrix is unitary.

While in ref. [4] the analysis was (within the stated assumptions) model-independent, here only the formalism is the same for all models. We introduce this formalism in section 2. To get numerical results we have to separately discuss sequential and non-sequential extra down quarks. We discuss models with extra down quarks in vector-like representations in section 3 and analyze the four generation model in section 4. We summarize our results in section 5.

2 Violation of CKM Unitarity and $B - \bar{B}$ Mixing

We consider models where the new physics does not contribute significantly to W-mediated tree level processes. Most well-motivated extensions of the SM belong to this class. The SM-dominance of these decays has three relevant consequences:

(i) The phase of the $b \to c\bar{c}s$ decay amplitude, $A_{c\bar{c}s}$, is the CKM phase, $\arg(V_{cb}V^*_{cs})$.

(ii) The absorptive part of the $B - \bar{B}$ mixing amplitude is not significantly modified by the new physics, $\Gamma_{12} \approx \Gamma^{\rm SM}_{12}$.

(iii) The following measurements of CKM parameters are valid in our framework [5]:

$$\begin{aligned} |V_{ud}| &= 0.9740 \pm 0.0010, & |V_{cd}| &= 0.224 \pm 0.016, \\ |V_{us}| &= 0.2196 \pm 0.0023, & |V_{cs}| &= 1.04 \pm 0.16, \\ |V_{ub}/V_{cb}| &= 0.08 \pm 0.02, & |V_{cb}| &= 0.0395 \pm 0.0017. \end{aligned} \tag{3}$$

The ranges in eq. (3) lead to the following bound, which plays a role in our discussion below:

$$R_u \equiv \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| \lesssim 0.47. \tag{4}$$

We note that for processes where the SM tree and penguin contributions are comparable, such as $b \to u\bar{u}s$ decays, the new physics contributions could be significant. This is the reason that we do not use the bounds on the angle γ of the unitarity triangle that follow from $B \to \pi K$ decays [6].

Our investigation concerns models where the 3×3 CKM matrix is not unitary. In particular, we are interested in violation of the relation $V_{ud}^*V_{ub} + V_{cd}^*V_{cb} + V_{td}^*V_{tb} = 0$. In any given model of this class, we can define a quantity U_{db} such that

$$V_{ud}^*V_{ub} + V_{cd}^*V_{cb} + V_{td}^*V_{tb} - U_{db} = 0. \tag{5}$$

The physical interpretation of U_{db} will be model-dependent. However, in all models it gives a useful parameterization of both the new physics contribution to $B - \bar{B}$ mixing and the violation of CKM unitarity. In particular, it is convenient to discuss the violation of CKM unitarity in terms of the *unitarity quadrangle* drawn in fig. 1.

We find it convenient to define also the following quantity (see fig. 1 for its geometrical interpretation):

$$X_{db} = U_{db}^* - V_{td}V_{tb}^*. \tag{6}$$

We can bound $|X_{db}|$ through

$$|V_{cd}V_{cb}^*|_{\min} - |V_{ud}V_{ub}^*|_{\max} \leq |X_{db}| \leq |V_{cd}V_{cb}^*|_{\max} + |V_{ud}V_{ub}^*|_{\max}. \tag{7}$$

Using eq. (3) we get

$$0.004 \lesssim |X_{db}| \lesssim 0.014. \tag{8}$$

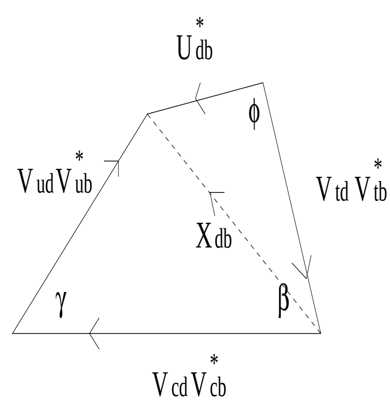


Figure 1: The unitarity quadrangle.

The CP asymmetry in $B \to \psi K_S$ depends on the relative phase between the $B - \bar{B}$ mixing amplitude and the $b \to c\bar{c}s$ decay amplitude. In our framework, neither $A_{c\bar{c}s}$ nor Γ_{12} are significantly affected by the new physics. However, the new physics may give significant contributions to the dispersive part of the mixing amplitude, M_{12}. The modification of M_{12} can be parameterized as follows (see for example [7, 8]):

$$M_{12} = r_d^2 e^{2i\theta_d} M_{12}^{\rm SM}. \tag{9}$$

At present we have two experimental probes of M_{12}. The mass difference between the two neutral B mesons, Δm_B, is given by

$$\Delta m_B = 2 r_d^2 |M_{12}^{\rm SM}|, \tag{10}$$

so that its experimental value gives:

$$7.1 \times 10^{-3} \le r_d |V_{td} V_{tb}^*| \le 10.75 \times 10^{-3}. \tag{11}$$

The CP asymmetry in $B \to \psi K_S$, $a_{\psi K_S}$, is given by

$$a_{\psi K_S} = \sin[2(\beta + 2\theta_d)], \tag{12}$$

so that its experimental value gives:

$$\sin[2(\beta + \theta_d)] \gtrsim \begin{cases} 0.35 & \text{one sigma,} \\ 0 & 95\% \text{ CL.} \end{cases} \tag{13}$$

The CDF measurement constrains the combination $\beta + \theta_d$ through eq. (13). We are interested in finding whether the range of the phase θ_d is constrained. We find that this is indeed the case in models where the following situation holds: First, there should be an independent upper bound on β, $|\beta| \le \beta_{\rm max}$. Second, this upper bound should be strong enough in the sense that the following inequality holds:

$$\beta_{\rm max} < \frac{1}{2} \left(\arcsin(a_{\psi K_S}) + \frac{\pi}{2} \right). \tag{14}$$

Then, the following limit on $\sin 2\theta_d$ holds:

$$\sin(2\theta_d) > \sin(\arcsin(a_{\psi K_S}) - 2\beta_{\rm max}). \tag{15}$$

If CKM unitarity is not violated, then $\beta_{\rm max} \approx \pi/6$, leading to $\sin 2\theta_d \gtrsim -0.6(-0.87)$ at one sigma (95% CL) [4]. If, however, CKM unitarity is violated, then there is no model-independent constraint on $|V_{td} V_{tb}^*|$ and/or β and we cannot constrain r_d or θ_d without further input. We will constrain r_d and θ_d in the framework of specific models in the next two sections.

The r_d and θ_d parameters are related also to other physical observables. For instance, the CP asymmetry in semileptonic decays, $a_{\rm SL}$, is given by:

$$a_{\rm SL} = \mathcal{I}m \frac{\Gamma_{12}}{M_{12}}. \tag{16}$$

Since $(\Gamma_{12}/M_{12})^{\rm SM}$ is real to a good approximation, the effects of new physics within our framework can be written as follows (for more details see ref. [4]):

$$\frac{a_{\rm SL}}{(\Gamma_{12}/M_{12})^{\rm SM}} = -\frac{\sin 2\theta_d}{r_d^2}. \tag{17}$$

Our analysis allows us to constrain $a_{\rm SL}$ within these specific models.

As mentioned above, the special point about extensions of the quark sector is that the violation of CKM unitarity and the new contributions to $B-\bar{B}$ mixing are related. Specifically, if we define

$$re^{i\phi} \equiv \frac{U_{db}^*}{V_{tb}^* V_{td}}, \tag{18}$$

then we have, in general,

$$r_d^2 e^{2i\theta_d} = 1 + are^{-i\phi} - br^2 e^{-2i\phi}. \tag{19}$$

The a and b parameters are model-dependent.

To understand the consequences of eqs. (18) and (19), note that eq. (8) gives bounds on

$$|X_{db}| = (1 + r^2 - 2r\cos\phi)^{1/2} |V_{td}V_{tb}^*|, \tag{20}$$

while eq. (11) gives bounds on

$$r_d |V_{td}V_{tb}^*| = (1 + 2ar\cos\phi - 2br^2\cos 2\phi - 2abr^3\cos\phi + a^2r^2 + b^2r^4)^{1/4} |V_{td}V_{tb}^*|. \tag{21}$$

The fact that the two constraints have to be satisfied for the same value of $|V_{td}V_{tb}^*|$ may exclude regions in the (ϕ, r) plane.

3 Extra SU(2)-Singlet Down Quarks

We consider a model with extra down quarks in a vector-like representation of the SM gauge group, $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. In addition to the three quark generations, each consisting of the three representations

$$Q_{Li}(3,2)_{+1/6}, \quad u_{Ri}(3,1)_{+2/3}, \quad d_{Ri}(3,1)_{-1/3}, \quad (i = 1,2,3), \tag{22}$$

we have the following vector-like representation:

$$d_4(3,1)_{-1/3} \; + \; \bar{d}_4(\bar{3},1)_{+1/3}. \tag{23}$$

Such quark representations appear, for example, in E_6 GUTs.

The most important feature of this model to our purposes is that it allows for flavor changing $Z\bar{d}b$-couplings (for details see refs. [9]-[15]):

$$\mathcal{L}_{Zdb} = -\frac{g}{2\cos\theta_W} U_{db} Z_\mu \bar{d}_L \gamma^\mu b_L + {\rm h.c.}. \tag{24}$$

The U_{db} mixing parameter in eq. (24) is the same parameter defined in eq. (5) which signifies violation of CKM unitarity. It allows Z-mediated tree level contributions to flavor changing neutral current processes such as $B \to \mu^+\mu^- X_d$. The experimental bound on the rate of this decay gives (see *e.g.* [16] and references therein):

$$|U_{db}/V_{cb}| \leq 0.04 \quad \to \quad |U_{db}| \leq 0.0016. \tag{25}$$

Putting the bound on $|U_{db}|$ of eq. (25) and the values of the CKM parameters of eq. (3) into

$$|V_{cd}V_{cb}^*|_{\min} - |V_{ud}V_{ub}^*|_{\max} - |U_{db}|_{\max} \leq |V_{td}V_{tb}^*| \leq |V_{cd}V_{cb}^*|_{\max} + |V_{ud}V_{ub}^*|_{\max} + |U_{db}|_{\max}, \tag{26}$$

we get

$$2.7 \times 10^{-3} \leq |V_{td}V_{tb}^*| \leq 1.6 \times 10^{-2}. \tag{27}$$

Using eqs. (27) and (11), we can constrain r_d:

$$0.44 \lesssim r_d \lesssim 4.1. \tag{28}$$

Putting eqs. (25) and (3) into

$$|\sin\beta| \leq \frac{|V_{ud}V_{ub}^*|_{\max} + |U_{db}|_{\max}}{|V_{cd}V_{cb}^*|_{\min}} = (R_u)_{\max} + \frac{1}{|V_{cd}|_{\min}} \left|\frac{U_{db}}{V_{cb}^*}\right|_{\max}, \tag{29}$$

we get

$$\beta_{\max} \approx \frac{2\pi}{9}. \tag{30}$$

Using eqs. (30) and (13), we can constrain θ_d:

$$\sin 2\theta_d \gtrsim \begin{cases} -0.88 & \text{one sigma,} \\ -0.99 & 95\% \text{ CL.} \end{cases} \tag{31}$$

In the derivation of (28) and (31), we have not used the correlation between violation of CKM unitarity and contribution to $B-\bar{B}$ mixing. To do so, we note that the U_{db} coupling of eq. (24) allows a Z-mediated tree diagram contribution to $B-\bar{B}$ mixing. It is possible to parameterize the new contributions to M_{12} as in eq. (19) [15, 17, 18], with

$$a = \frac{4\bar{C}(x_t)}{\bar{E}(x_t)}, \quad b = \frac{4\pi \sin^2\theta_W}{\alpha \bar{E}(x_t)}. \tag{32}$$

Here $x_t = (m_t/M_W)^2$ and $\bar{C}(x_t)$ and $\bar{E}(x_t)$ are the Inami-Lim functions [19]:

$$\bar{E}(x_t) = \frac{-4x_t + 11x_t^2 - x_t^3}{4(1-x_t)^2} + \frac{3x_t^3 \ln x_t}{2(1-x_t)^3}, \tag{33}$$

$$\bar{C}(x_t) = \frac{x_t}{4}\left[\frac{4-x_t}{1-x_t} + \frac{3x_t \ln x_t}{(1-x_t)^2}\right]. \tag{34}$$

An explicit calculation [17] gives $a = -3.3$ and $b = -160$.

We now relate all of our constraints to the (ϕ, r) parameters by performing the following procedure. We scan the $\phi - r$ parameter space. For each (ϕ, r) pair we calculate r_d and θ_d through eq. (19) and $|X_{db}/(V_{td}V_{tb}^*)|$ through eq. (20). We check whether there exist values of $|V_{td}V_{tb}^*|$ that are consistent with all the constraints in eqs. (8), (11), (18) and (25). Note that at this stage we do not yet incorporate the $a_{\psi K_S}$ constraint. The allowed region is displayed in fig. 2(a). The upper bound on r is a result of inconsistency between the upper bound of eq. (11) and the lower bound of eq. (8). The small excluded regions at $\cos\phi \approx 0$ correspond to $r \approx |b|^{-1/2}$, where r_d is too small.

Next we incorporate the constraint from $a_{\psi K_S}$. For each pair of values (ϕ, r), we calculate $\beta_{\rm max}$:

$$\beta_{\rm max} = \left|\arcsin\left(\frac{r\sin\phi}{(1-2r\cos\phi+r^2)^{1/2}}\right)\right| + \left[\arccos\left(\frac{|V_{cd}V_{cb}^*|^2 + |X_{db}|^2 - |V_{ud}V_{ub}^*|^2}{2|V_{cd}V_{cb}^*||X_{db}|}\right)\right]_{\rm max}. \tag{35}$$

If the condition in eq. (14) holds (and $r\cos\phi < 1$), we exclude (ϕ, r) pairs that violate the bound in eq. (15). The allowed parameter space is displayed in fig. 2(b).

Our numerical scan gives, at the one sigma level:

$$-0.8 \lesssim \sin 2\theta_d \leq 1, \tag{36}$$

$$0.5 \lesssim r_d \lesssim 3.2. \tag{37}$$

For the semileptonic asymmetry we find (see eq. (17)):

$$-4.0 \lesssim \frac{a_{SL}}{(\Gamma_{12}/M_{12})^{SM}} \lesssim 1.4. \tag{38}$$

4 A Fourth Generation

The second model we consider is the SM with a fourth sequential generation. Here the quark content is as given in eq. (22) with $i = 1, 2, 3, 4$. A fourth generation by itself is now excluded by electroweak precision data [5]. However, if there is new physics in addition to a fourth generation, such that the electroweak precision data constraints are relaxed but M_{12} is not affected by this extra new physics, then our analysis below applies. The analysis in this section applies also to a model in which extra up quarks in a vector-like representation,

$$u_4(3,1)_{+2/3} \ + \ \bar{u}_4(\bar{3},1)_{-2/3}, \tag{39}$$

are added to the SM three generations [15].

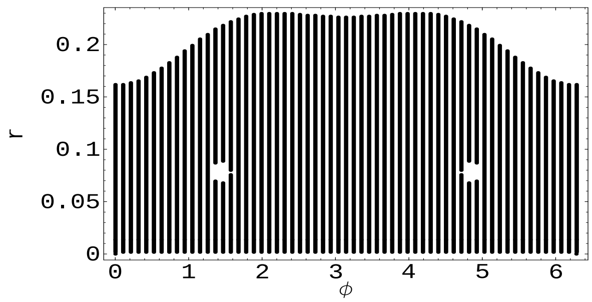
Within these models,

$$U_{db} = -V_{t'd}^* V_{t'b}. \tag{40}$$

From unitarity of the 4×4 matrix, we have [20, 21]:

$$|V_{td}^* V_{tb}| \leq 0.1, \qquad |U_{db}| \leq 0.1. \tag{41}$$

(a)



(b)

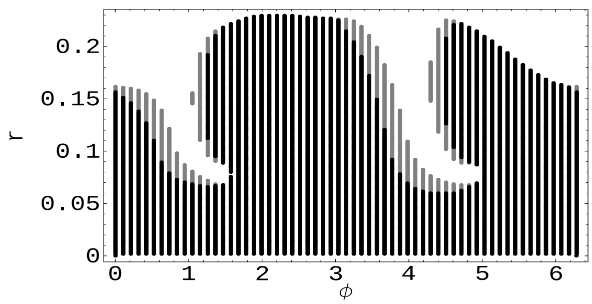


Figure 2: SU(2)-singlet down quarks. (a) The Δm_B constraint. The dark region is allowed. (b) The combination of the Δm_B and $a_{\psi K_S}$ constraints. The dark (light plus dark) grey region is the allowed region corresponding to the one sigma (95% CL) bound, $a_{\psi K_S} \geq 0.35$ (0).

The reasonable agreement of the most recent data on $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ and the expectation of the three generation SM implies that [7]

$$1 \simeq |V_{tb}|^2 + |V_{t'b}|^2 \left(\frac{m_{t'}}{m_t}\right)^2 . \tag{42}$$

For $m_{t'} = 500\ GeV$, eq. (42) leads to

$$|U_{db}| \lesssim 0.03. \tag{43}$$

In our analysis, we use the bound (43).

The analysis goes along similar lines to that of the previous section. However, the fact that eq. (43) gives $|U_{db}|_{\max} > |V_{cd}V_{cb}^*|_{\min}$ means that we can put neither a meaningful lower bound on $|V_{td}V_{tb}^*|$ nor a meaningful upper bound on $|\sin\beta|$. Instead of (27), we now have

$$|V_{td}V_{tb}^*| \leq 0.044, \tag{44}$$

and, consequently, eq. (11) gives only a lower bound on r_d:

$$r_d \gtrsim 0.16. \tag{45}$$

As concerns $\sin 2\theta_d$, we get no bounds.

Next, we want to incorporate the relation between violation of CKM unitarity and new contributions to $B-\bar{B}$ mixing. In the four generation model, the new contributions to $B-\bar{B}$ mixing come from box diagrams involving t' quarks. These contributions can again be parameterized as in eq. (19) [22, 23] with

$$a = -2\frac{\bar{E}(x_{t'}, x_t)}{\bar{E}(x_t)}, \quad b = -\frac{\bar{E}(x_{t'})}{\bar{E}(x_t)}, \tag{46}$$

$$\begin{aligned} \bar{E}(x_{t'}, x_t) \;&=\; -x_{t'}x_t\left[\frac{1}{x_{t'}-x_t}\left(\frac{1}{4}-\frac{3}{2}\frac{1}{(x_{t'}-1)}-\frac{3}{4}\frac{1}{(x_{t'}-1)^2}\right)\ln x_{t'}\right. \\ &\left. + (x_{t'} \leftrightarrow x_t) - \frac{3}{4}\frac{1}{(x_{t'}-1)(x_t-1)}\right], \end{aligned} \tag{47}$$

and $\bar{E}(x)$ defined in eq. (33). Taking $m_t \approx 170\ GeV$ and $180\ GeV \lesssim m_{t'} \lesssim 500\ GeV$, we find:

$$-4 \lesssim a \lesssim -2, \qquad\qquad -5.5 \lesssim b \lesssim -1. \tag{48}$$

Below we display only the results of a numerical analysis for the case $m_{t'} = 500\ GeV$ for which the effects are most significant. In this case: $a \sim -3.8$ and $b \sim -5.4$. The allowed region without the $a_{\psi K_S}$ constraint is given in fig. 3(a). We only display the $r < 1$ region since the $a_{\psi K_S}$ constraint will have no effect for $r > 1$. The excluded region around $\phi = 0$ is a result of inconsistency between the upper bound of eq. (11) and the lower bound of eq. (8). The small excluded regions at $r \approx 0.4$ correspond to r_d values that are too small. Note that r_d can be very large in this model, corresponding to a nearly vanishing $|V_{td}V_{tb}^*|$ and, consequently, nearly vanishing $M_{12}^{\rm SM}$. Incorporating the $a_{\psi K_S}$ constraint, we find that the new CDF measurement does not place significant new constraints on the parameter space of the four generation model. This is particularly true for a t'-mass that is not much higher than m_t. The allowed region is displayed in fig. 3(b).

Our numerical scan gives

$$r_d \gtrsim 0.33, \tag{49}$$

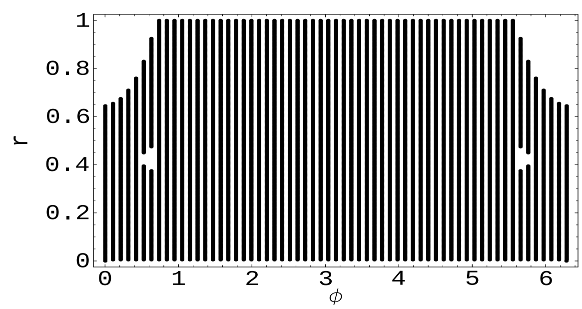
and no bounds on $\sin 2\theta_d$. For the semileptonic asymmetry, we get

$$-9.0 \lesssim \frac{a_{\rm SL}}{(\Gamma_{12}/M_{12})^{\rm SM}} \lesssim 6.1. \tag{50}$$

5 Conclusions

In a previous work with Barenboim [4], we used the new CDF measurement of the CP asymmetry in $B \to \psi K_S$ to derive the first constraint on the phase of new physics contributions to $B-\bar{B}$ mixing. We have done so in the framework of models where the CKM

(*a*)



(*b*)

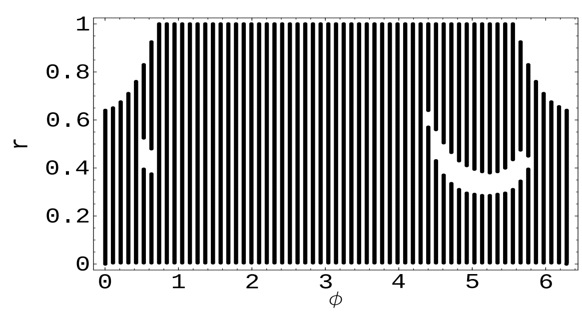


Figure 3: Four generations, $m_{t'} = 500\ GeV$. The dark region is allowed. (a) The Δm_B constraint. (b) The combination of the Δm_B and $a_{\psi K_s}$ one sigma constraints.

matrix is unitary. In this work we have shown that significant constraints apply also in extensions of the quark sector, where the 3×3 CKM matrix is not unitary. The main reason that makes this possible is that a single complex parameter (U_{db}) characterizes both the violation of CKM unitarity and the new contributions to $B - \bar{B}$ mixing. Therefore, the number of relevant new parameters is effectively the same as in models where the CKM matrix is unitary. In either case, the measurement of $a_{\psi K_S}$ gives the first constraint on the phase $2\theta_d = \arg(M_{12}/M_{12}^{\rm SM})$. Specifically, whenever we can put an upper bound on $|\beta|$ that is lower than $\frac{1}{2}\left(\arcsin(a_{\psi K_S}) + \frac{\pi}{2}\right)$, it follows that there is a lower bound on $\sin 2\theta_d$.

Our most significant results concern models with extra SU(2)-singlet down quarks. The constraints on the relevant mixing parameters are displayed in fig. 2(b). In particular, the measurement of $a_{\psi K_S}$ gives $\sin 2\theta_d \gtrsim -0.8$. This phase is related to the phase of U_{db} which, in this framework, parametrizes the flavor changing $Z\bar{d}b$ coupling. The bound on $\sin 2\theta_d$ together with constraints from Δm_B give bounds on the CP asymmetry in semileptonic B decays, $-1.4 \times 10^{-2} \lesssim a_{\rm SL} \lesssim 4.0 \times 10^{-2}$. Weaker constraints apply to the four generation model and to models with extra up quarks in vector-like representations.

Acknowledgements: We thank JoAnne Hewett for useful discussions. Y.N. is supported

in part by the United States – Israel Binational Science Foundation (BSF), by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities and by the Minerva Foundation (Munich).

References

[1] CDF Collaboration, CDF/PUB/BOTTOM/CDF/4855 (1999).

[2] K. Ackerstaff et al., Eur. Phys. J. C5 (1998) 379, hep-ex/9801022.

[3] F. Abe et al., Phys. Rev. Lett. 81 (1998) 5513, hep-ex/9806025.

[4] G. Barenboim, G. Eyal and Y. Nir, hep-ph/9905397.

[5] C. Caso et al., Eur. Phys. J. C3 (1998) 1.

[6] M. Neubert, JHEP 02 (1999) 014, hep-ph/9812396; hep-ph/9904321.

[7] The BaBar Physics Book, eds. P.F. Harrison and H.R. Quinn, SLAC-R-504 (1998).

[8] Y. Grossman, Y. Nir and M.P. Worah, Phys. Lett. B407 (1997) 307, hep-ph/9704287.

[9] Y. Nir and D. Silverman, Nucl. Phys. B345 (1990) 301.

[10] Y. Nir and D. Silverman, Phys. Rev. D42 (1990) 1477.

[11] D. Silverman, Phys. Rev. D45 (1992) 1800; Int. J. Mod. Phys. A11 (1996) 2253, hep-ph/9504387; Phys. Rev. D58 (1998) 095006, hep-ph/9806489.

[12] G.C. Branco *et al.*, Phys. Rev. D48 (1993) 1167.

[13] W.-S. Choong and D. Silverman, Phys. Rev. D49 (1994) 2322.

[14] L. Lavoura and J.P. Silva, Phys. Rev. D47 (1993) 1117.

[15] V. Barger, M.S. Berger and R.J.N. Phillips, Phys. Rev. D52 (1995) 1663, hep-ph/9503204.

[16] Y. Grossman, Y. Nir and R. Rattazzi, in *Heavy Flavours II*, eds. A.J. Buras and M. Lindner (World Scientific, Singapore), hep-ph/9701231.

[17] G. Barenboim and F.J. Botella, Phys. Lett. B433 (1998) 385, hep-ph/9708209.

[18] G. Barenboim, F.J. Botella, G.C. Branco and O. Vives, Phys. Lett. B422 (1998) 277, hep-ph/9709369.

[19] T. Inami and C.S. Lim, Prog. Theor. Phys. 65 (1981) 297; ibid 65 (1981) 1772(E).

[20] F.J. Botella and L.-L. Chau, Phys. Lett. B168 (1986) 97.

[21] H. Harari and M. Leurer, Phys. Lett. B181 (1986) 123.

[22] S. Wakaizumi, T. Hasuike, T. Hattori and T. Hayashi, Phys. Rev. D44 (1991) 3582.

[23] T. Hattori, T. Hasuike and S. Wakaizumi, hep-ph/9804412.